The Creation of the Particle Number Asymmetry in the Universe (Work based on ArXiv:1609:02990.)

Takuya Morozumi

Hiroshima University, Core-U

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Apriadi Salim Adam (Hiroshima Univ.)

Keiko Nagao

(National Institute of Tech., Niihama College)

Hiroyuki Takata(Tomsk Pedagogical Univ.)

Outline

1. The origin generating particle number asymmetry has not been fixed yet despite of many interesting scenarios.

2-1 We study a simple model generating particle number asymmetry through "interaction". The particle number is related to the global U(1) charge carried by a complex scalar field.

2-2 The interaction has the feature of CP violation and particle number violation.

3-1. **We compute the time evolution of the asymmetry using quantum field theory with the density matrix.**

3-2. **We show the formulation for the FRW universe with arbitrary time dependent scale factor ($a(x^0)$). The numerical result for the universe with constant scale factor is also shown.**

Model of scalars with particle number violation

N; Neutral scalar ϕ complex scalar

$$S = \int d^4x\sqrt{-g}(\mathcal{L}_{\rm free} + \mathcal{L}_{\rm int.}),$$

$$\mathcal{L}_{\rm free} = g^{\mu\nu}\nabla_\mu\phi^*\nabla_\nu\phi + \frac{B^2}{2}(\phi^2 + \phi^{*2}) - m_\phi^2|\phi|^2$$

$$+(\frac{\alpha_2}{2}\phi^2 + h.c.)R + \alpha_3|\phi|^2R$$

$$+\frac{g^{\mu\nu}}{2}(\nabla_\mu N\nabla_\nu N - m_N^2N^2)$$

$$\mathcal{L}_{\rm int.} = A\phi^2N + A^*\phi^{*2}N + A_0|\phi|^2N$$

One choose the metrix $g_{\mu\nu}$as;

$$g_{\mu\nu} = (1, -a(x^0)^2, -a(x^0)^2, -a(x^0)^2).$$

One introduces the real fields $\phi_i (i = 1 \sim 3)$;

$$\phi = \frac{\phi_1 + i\phi_2}{\sqrt{2}}, N = \phi_3.$$

The masses for them are given as,

$$\tilde{m}_3^2(x^0) = m_N^2$$

$$\tilde{m}_1^2(x^0) = m_\phi^2 - B^2 - (\alpha_2 + \alpha_3)R(x^0)$$

$$\tilde{m}_2^2(x^0) = m_\phi^2 + B^2 + (\alpha_2 - \alpha_3)R(x^0)$$

$$R(x^0) = 6\left(\frac{\ddot{a}}{a} + \left(\frac{\dot{a}}{a}\right)^2\right)$$

In terms of real fields ϕ_i $(i = 1 \sim 3)$, Lagrangian is,

$$\mathcal{L}_{\text{free+int}} = \sum_{i=1}^{3} \frac{1}{2}(g^{\mu\nu}\nabla_\mu\phi_i\nabla_\nu\phi_i - \tilde{m}_i^2(x^0)\phi_i^2) + \frac{1}{3}A_{ijk}\phi_i\phi_j\phi_k + O(\lambda_4\phi_i^4)$$

A_{ijk} is a symmmetric tensor and its non-zero components are

$$A_{113} = \frac{A_0}{2} + Re(A), A_{223} = \frac{A_0}{2} - Re(A),$$
$$A_{123} = -Im(A)$$

Particle number violating coupling

$$\rightarrow A_{113} - A_{223} \sim 2Re(A)$$

CP violating and Particle number violating coupling

$$\rightarrow A_{123} = -Im(A)$$

The particle number related to U(1) transformation

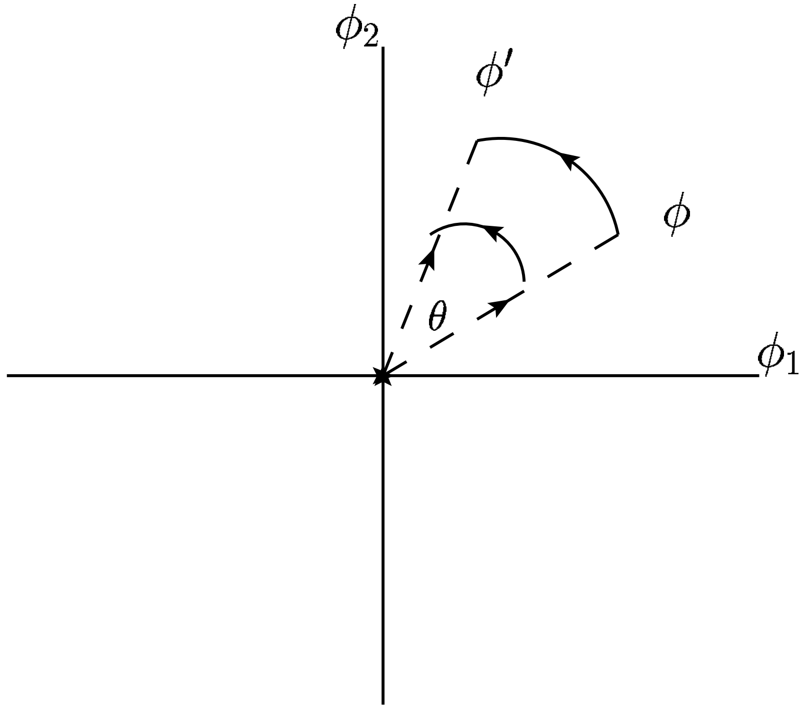


Figure 1: $\phi_1' = \phi_1 \cos\theta - \phi_2 \sin\theta$, $\phi_2' = \phi_2 \cos\theta + \phi_1 \sin\theta$.

The particle number related to U(1) transformation

- **U(1) transformation for the complex scalar field.**
  $\phi(x) \to e^{i\theta}\phi(x)$
- **U(1) charge** (eg.,Affleck and Dine (85) in susy)
  = particle number- anti-particle number

$$Q(x^0) = \int \sqrt{-g(x^0)}\, j_0(x) d^3\mathbf{x}$$

$$j_\mu(x) = \frac{1}{2}\phi_2 \overleftrightarrow{\partial}_\mu \phi_1 - \frac{1}{2}\phi_1 \overleftrightarrow{\partial}_\mu \phi_2.$$

How to compute the time evolution of particle number density

- setting the initial condition for the density matrix and expectation value of the fields

$$\rho(t_0) = \frac{e^{-\beta H_0}}{\mathrm{Tr} e^{-\beta \mathrm{H}_0}}, \quad \beta = \frac{1}{T}$$

$$H_0 = \text{Free Hamiltonian}$$

- Compute the expectation value of the particle number density through,

$$\langle j_0(x^0)\rho(t_0)\rangle = \mathrm{Tr}(j_0(x^0)\rho(t_0)).$$

The density matrix element is given as Euclidean classical action.

$$\langle \phi_i^1 | \rho | \phi_i^2 \rangle = \frac{e^{-S_{Ecl}^{\rm free}[\phi^1, \phi^2]}}{\int d\phi e^{-S_{Ecl}^{\rm free}[\phi, \phi]}}$$

with $\beta = 1/T$ (R. Hotta, H. Takata, T.M. PRD90, 016008)

$$S_{Ecl}^{\rm free} = \frac{1}{2} \sum_{i=1}^{3} \int_0^\beta du \Big[\left(\frac{\partial \phi_i}{\partial u} \right)^2 + \frac{1}{a_0^2} (\nabla \phi_i)^2 + \tilde{m}_{i0}^2 {\phi_i}^2 \Big]$$

$$\left. \frac{\delta S_{Ecl}^{\rm free}}{\delta \phi} \right|_{\phi = \phi_{cl}} = 0,$$

$$\phi(u = 0) = \phi^2, \phi(u = \beta) = \phi^1.$$

$$S_{Ecl}^{\rm free}[\phi^1, \phi^2] = S_{Ecl}^{\rm free}[\phi = \phi_{cl}]$$

Current expection value in terms of Green functions and expectation values for fields

$$j_\mu = \frac{1}{2}\left(\phi_2 \overleftrightarrow{\partial}_\mu \phi_1 - \phi_1 \overleftrightarrow{\partial}_\mu \phi_2\right)$$

Expectaion value with $\rho(t^0)$ (initial density matrix);

$$\langle j_\mu(x)\rangle = \mathrm{Tr}[j_\mu(x)\rho(t^0)]$$
$$= \lim_{y\to x}\left(\frac{\partial}{\partial x^\mu} - \frac{\partial}{\partial y^\mu}\right)\mathrm{Re.}[G_{12}(x,y)]$$
$$+ \mathrm{Re.}\left(\bar{\phi}_2^* \overleftrightarrow{\partial}_\mu \bar{\phi}_1\right)$$

$G_{ij}(x,y) = \mathrm{Tr}[\Phi_j(y)\rho(t^0)\Phi_i(x)]$ and
$\bar{\phi}_i = \mathrm{Tr}[\Phi_i(x)\rho(t^0)]$ are obtained from effective action.

2 Particle Irreducible effective action for Green functions and expectation values for fields

$$\Gamma[G,\bar\phi] = S[\bar\phi] + \frac{i}{2}\mathrm{TrLnG}^{-1}$$

$$+\frac{1}{2}\int d^4x d^4y \frac{\delta^2 S}{\delta\bar\phi_i^a(x)\delta\bar\phi_j^b(y)} G_{ij}^{ab}(x,y)$$

$$+\frac{i}{3}D_{abc}A_{ijk}\int\int d^4x d^4y\sqrt{-g(x)}\sqrt{-g(y)}$$

$$[G_{ii'}^{aa'}(x,y)G_{jj'}^{bb'}(x,y)G_{kk'}^{cc'}(x,y)]D_{a'b'c'}A_{i'j'k'}$$

Schwinger Dyson equations are solved perturbatively

$$G_{ij} = G_{ij}^{\mathrm{free}} + G_{ij}^{O(A)}, \quad \bar\phi_i = \bar\phi_{i\mathrm{free}} + \bar\phi_i^{O(A)}.$$

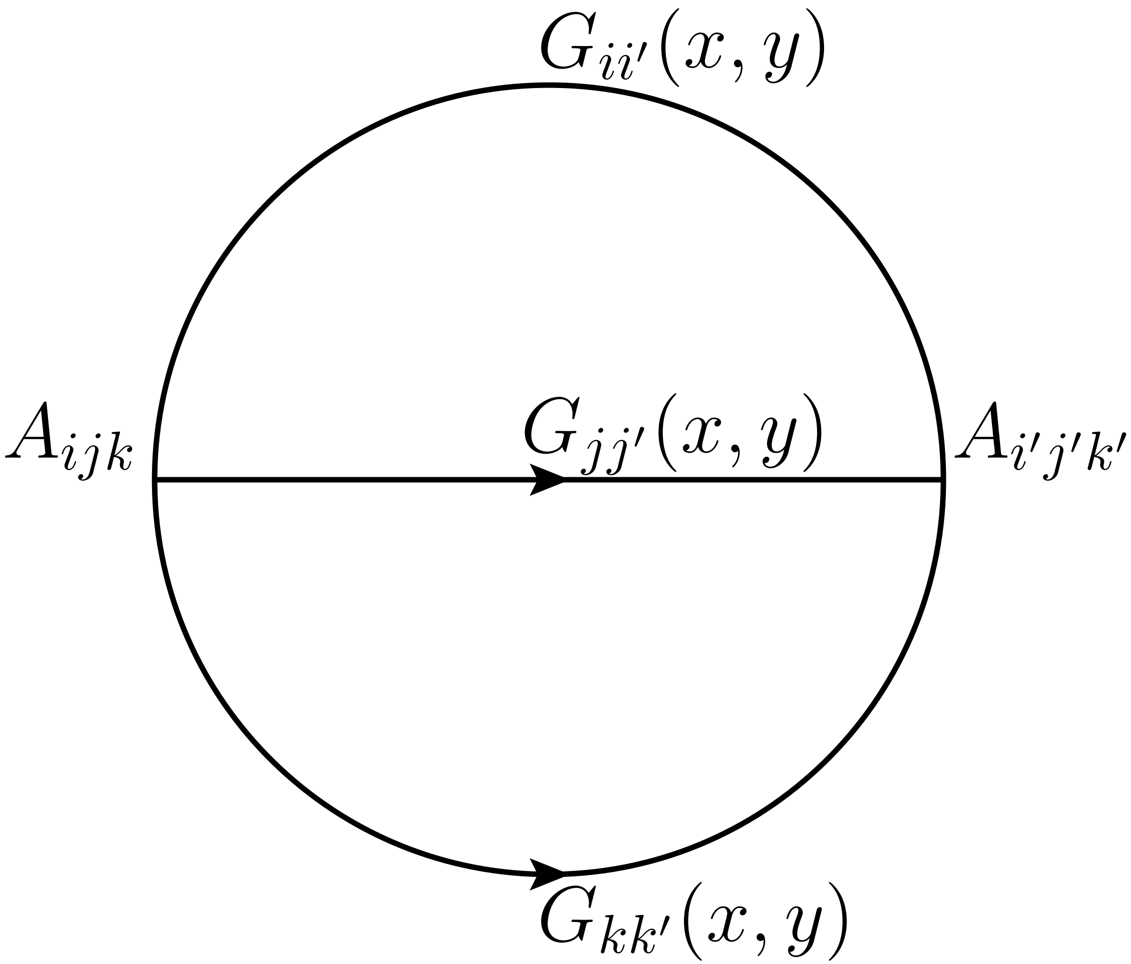


Figure 2: **2 Particle Irreducible graph**

Schwinger Dyson equation and Field equation. $G(x^0, y^0) = \left(\frac{a_0}{a(x^0)}\right)^{\frac{3}{2}} \hat{G}(x^0, y^0) \left(\frac{a_0}{a(y^0)}\right)^{\frac{3}{2}}, \bar{\phi} = \left(\frac{a_0}{a(x^0)}\right)^{\frac{3}{2}} \bar{\varphi}$.

$$\left(\frac{\partial^2}{\partial x^{02}} + \frac{\mathrm{k}^2}{a(x^0)^2} + \bar{m}_i^2(x^0)\right) \hat{G}_{ij}(x^0, y^0, \mathrm{k})$$

$$= 2(c \cdot ((D \otimes \hat{A}_{ilm} \cdot \bar{\varphi})_{mx^0}) \hat{G}_{mj}(x^0, y^0, \mathrm{k})$$

$$+O(A^2) + \text{boundary} \quad \text{terms.}$$

$$(\frac{\partial^2}{\partial x^{02}} + \bar{m}_i^2)\bar{\varphi}_i^d(x^0) = -i\delta_{x_0,t_0}\kappa_{ij}^{dc}c^{ce}{a_0}^3\bar{\varphi}(t_0)^e.$$

κ is given with the initial$(x^0 = t_0)$ density matrix.

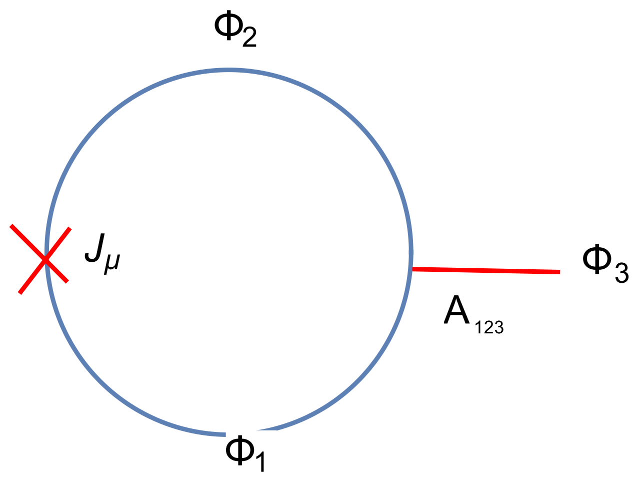


Figure 3: **The diagram corresponds $O(A)$ contribution**

The asymmmetry up to $O(A)$ for $a(x^0)$ =constant and $\bar{\phi}_1(0) = \bar{\phi}_2(0) = 0$.

$$\langle j_0(x^0)\rangle = \langle j_0(x^0)\rangle^{emission}_{absorption} + \langle j_0(x^0)\rangle^{inversedecay}_{decay} + \langle j_0(x^0)\rangle_{vacuum}$$

$$\langle j_0(x^0)\rangle^{emiss.}_{absorp.} = \frac{\overline{\phi_3}(0)A_{123}}{2a_0^3}\int \frac{d^3k}{(2\pi)^3}\left(\frac{1}{\omega_{1k}} + \frac{1}{\omega_{2k}}\right) \times$$

$$\Bigg[\Bigg\{\frac{\coth\frac{\beta\omega_{2k}}{2} - \coth\frac{\beta\omega_{1k}}{2}}{2} + \tanh\frac{\beta\omega_3}{2}\Bigg\}$$

$$\times\Bigg\{\frac{\sin(\omega_{2k} - \omega_{1k})x^0 + \sin\omega_3 x^0}{\omega_{2k} - \omega_{1k} + \omega_3}\Bigg\} - (1 \leftrightarrow 2)\Bigg]$$

Physical meaning of absorption and emission: When the denominator of the time dependent factor vanishes, it corresponds to the on-shell process. (Absorption and emission of "quantum" with $\omega_3 = m_3$)

$$\left\{\frac{\sin(\omega_2(k) - \omega_1(k))x^0 + \sin\omega_3 x^0}{\omega_2(k) - \omega_1(k) + \omega_3}\right\}$$

$$\omega_{1k} = \omega_{2k} + \omega_3 \quad m_1 > m_2 + \omega_3$$

$$\left\{\frac{\sin(\omega_1(k) - \omega_2(k))x^0 + \sin\omega_3 x^0}{\omega_1(k) - \omega_2(k) + \omega_3}\right\}$$

$$\omega_{2k} - \omega_{1k} = \omega_3 \quad m_2 > m_1 + \omega_3$$

$$\langle j_0(x^0)\rangle_{decay}^{inv.decay} =$$

$$\frac{\overline{\phi_3}(0)A_{123}}{2a_0^3}\int\frac{d^3k}{(2\pi)^3}\left(\frac{1}{\omega_{2k}}-\frac{1}{\omega_{1k}}\right)$$

$$\times\{\frac{\coth\frac{\beta\omega_{2k}}{2}+\coth\frac{\beta\omega_{1k}}{2}-\tanh\frac{\beta\omega_3}{2}}{2}\}$$

$$\times\{\frac{\sin\omega_3 x^0-\sin(\omega_{1k}+\omega_{2k})x^0}{\omega_3-\omega_{2k}-\omega_{1k}}\}.$$

The condition that the decay $(3\rightarrow 1+2)$ and inverse decay $(1+2\rightarrow 3)$ happen; $(\omega_{2k}+\omega_{1k}=\omega_3)$, $\omega_3>m_1+m_2$

$$\langle j_0(x^0)\rangle_{vacuum} = \langle j_0(x^0)\rangle_{decay}^{inv.decay}(\omega_{1,2k} \to -\omega_{1,2k})$$

$$= \frac{\overline{\phi_3}(0)A_{123}}{2a_0^3}\int \frac{d^3k}{(2\pi)^3}\left(\frac{1}{\omega_{2k}} - \frac{1}{\omega_{1k}}\right)$$

$$\times \{\frac{\coth\frac{\beta\omega_{2k}}{2} + \coth\frac{\beta\omega_{1k}}{2}}{2} + \tanh\frac{\beta\omega_3}{2}\}$$

$$\times \{\frac{\sin\omega_3 x^0 + \sin(\omega_{1k}+\omega_{2k})x^0}{\omega_3+\omega_{2k}+\omega_{1k}}\}$$

The denominator; $\omega_3 + \omega_{2k} + \omega_{1k}$ is positive definite and the process is always a virtual process.

$$\langle j_0(x^0)\rangle = J_1(x^0) + J_2(x^0) + J_3(x^0) + J_4(x^0)$$

$$\langle j_0(x^0)\rangle^{inv.decay}_{decay} + \langle j_0(x^0)\rangle_{vacuum} \equiv J_2(x^0) + J_3(x^0)$$

$$J_2(x^0) = -\frac{A_{123}\overline{\phi}_3(0)}{\omega_3}(m_1^2 - m_2^2)\tanh\frac{\beta\omega_3}{2}$$

$$\int \frac{k^2 dk}{2\pi^2\omega_{1k}\omega_{2k}} \frac{\frac{\sin\omega_3 x^0}{\omega_3} - \frac{\sin(\omega_{1k}+\omega_{2k})x^0}{\omega_{1k}+\omega_{2k}}}{1-(\frac{\omega_{1k}+\omega_{2k}}{\omega_3})^2}$$

$$J_3(x^0) = -\frac{A_{123}\overline{\phi}_3(0)}{\omega_3}(m_1^2 - m_2^2)$$

$$\int \frac{k^2 dk}{2\pi^2\omega_{1k}\omega_{2k}} \frac{\coth\frac{\beta\omega_{1k}}{2} + \coth\frac{\beta\omega_{2k}}{2}}{2}$$

$$\times \quad \frac{\frac{\sin(\omega_{1k}+\omega_{2k})x^0}{\omega_3} - \frac{\sin\omega_3 x^0}{\omega_{1k}+\omega_{2k}}}{1-(\frac{\omega_{1k}+\omega_{2k}}{\omega_3})^2}$$

$$\langle j_0(x^0)\rangle^{emission}_{absorption} \equiv J_1(x^0) + J_4(x^0).$$

$$J_1(x^0) = \frac{A_{123}\overline{\phi}_3(0)}{\omega_3}(m_1^2 - m_2^2)\tanh\frac{\beta\omega_3}{2}$$

$$\lim_{d\to 4}\int \frac{d^{d-1}k}{(2\pi)^{d-1}}\frac{1}{\omega_{1k}\omega_{2k}}\frac{\frac{\sin\omega_3 x^0}{\omega_3} - \frac{\omega_{1k}+\omega_{2k}}{m_1^2-m_2^2}\sin(\frac{m_1^2-m_2^2}{\omega_{1k}+\omega_{2k}}x^0)}{1-(\frac{m_2^2-m_1^2}{(\omega_{2k}+\omega_{1k})\omega_3})^2}$$

$$J_4(x^0) = A_{123}\overline{\phi}_3(0)\int\frac{k^2dk}{4\pi^2}\frac{(\omega_{1k}+\omega_{2k})^2}{\omega_{1k}\omega_{2k}}\Big[\frac{2e^{-\beta\omega_{2k}}}{1-e^{-\beta\omega_{2k}}} - (2\to 1)\Big]$$

$$\times\frac{-\frac{\omega_{1k}+\omega_{2k}}{\omega_3}\sin\omega_3 x^0 + \frac{m_1^2-m_2^2}{\omega_3^2}\sin(\frac{m_1^2-m_2^2}{\omega_{1k}+\omega_{2k}}x^0)}{(\omega_{1k}+\omega_{2k})^2 - (\frac{m_2^2-m_1^2}{\omega_3})^2}$$

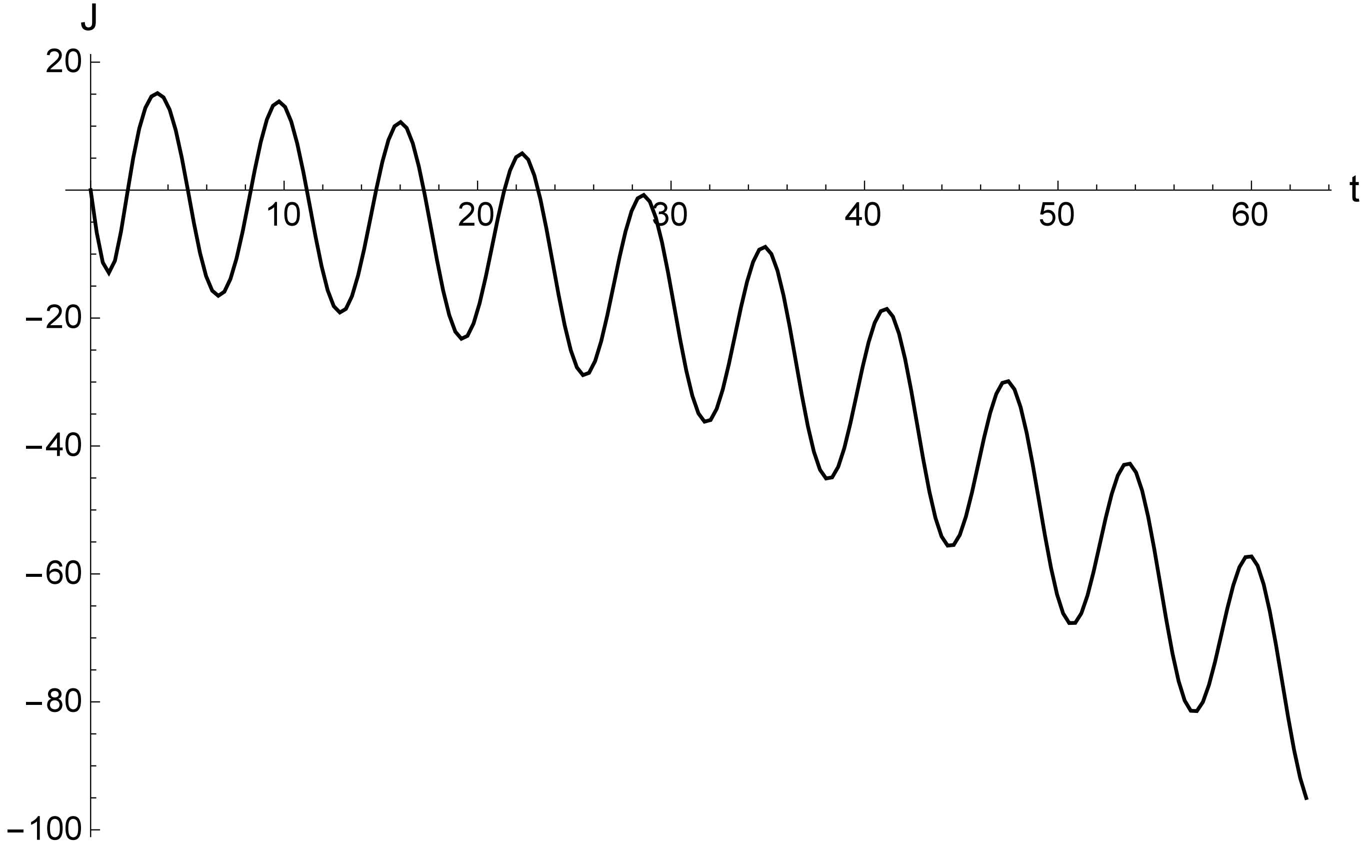


Figure 4: Time $t = \omega_3 x^0$ dependence of the asymmetry. $\omega_3 = 0.35, m_2 = 3, m_1 = 2, T = 100$

Figure 5: dependence on $\frac{m_2 - m_1}{\omega_3}$. $\frac{0.2}{0.35}$, $\frac{3-2}{0.35}$, $\frac{3}{0.35}$.

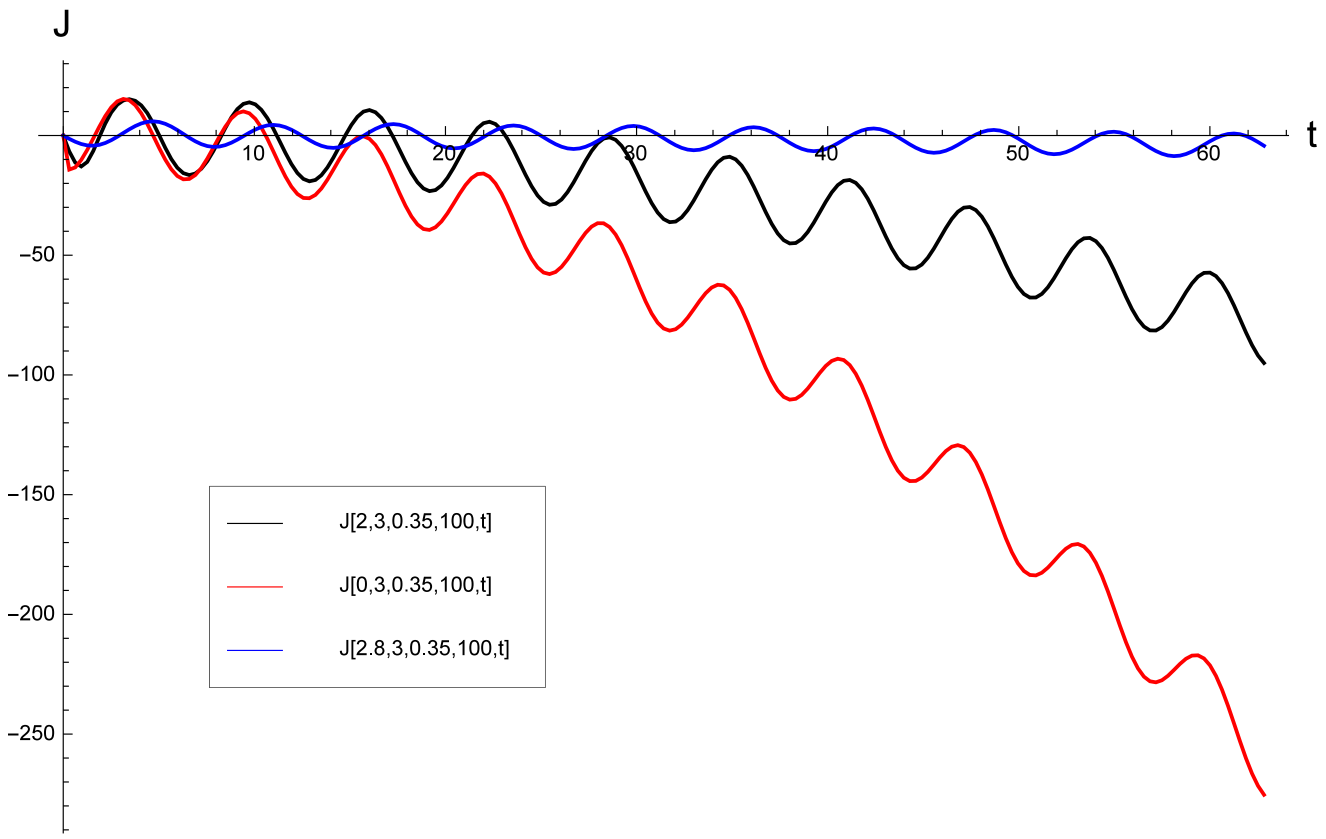
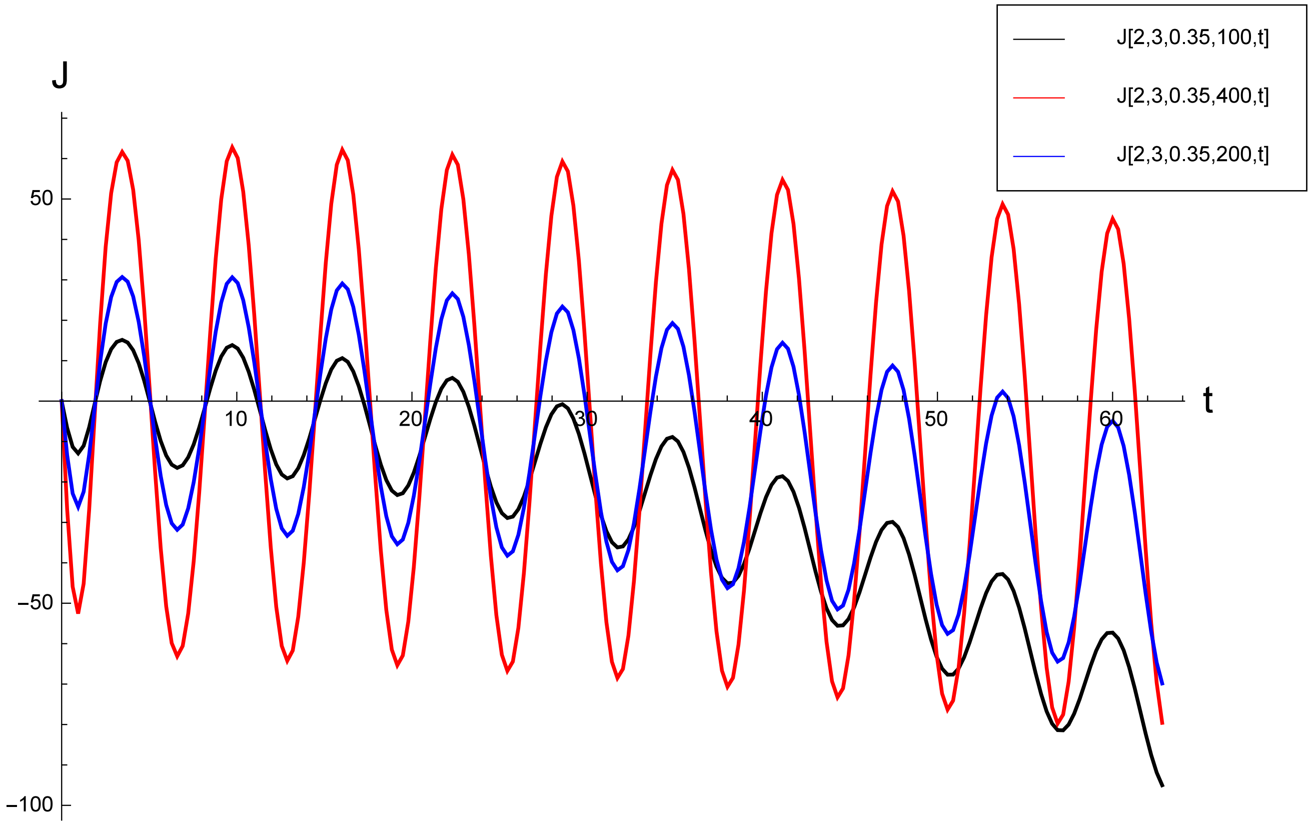


Figure 6: Temperature (T) dependence. $T = 100, 200, 400$.



Summary

- We have studied a simple model which generates the particle number asymmetry through interaction.
- We adopt the dendity matrix formulation (Calzetta and Hu) and calculate the time evolution of the current.
- We identified $O(A)$; (A is a coupling constant) contribution in which the field expectation value for a neutral scalar $\phi_3 = N$ and the Green function for the complex scalar involved.
- We numerically compute the asymmetry for the case with time-independent scale factor and show that the asymmetry is generated. The asymmetry is oscillatory growing with respect to time.
- The sign of the asymmetry is determined by the signs of the initial value $\bar{\phi}_3(0)$ and CP and U(1) breaking coupling constant A_{123}